**5.2.** Use the Fourier transform analysis equation (5.9) to calculate the Fourier transforms of:

**(a)** $\delta[n-1]+\delta[n+1]$ **(b)** $\delta[n+2]-\delta[n-2]$

Sketch and label one period of the magnitude of each Fourier transform.

(a) Let $x[n]=\delta[n-1]+\delta[n+1]$. Using the Fourier transform analysis equation (5.9), the Fourier transform $X(e^{j\omega})$ of this signal is

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\ &= e^{-j\omega}+e^{j\omega} = 2\cos\omega \end{aligned}$$

(b) Let $x[n]=\delta[n+2]-\delta[n-2]$. Using the Fourier transform analysis equation (5.9), the Fourier transform $X(e^{j\omega})$ of this signal is

$$\begin{aligned} X(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\ &= e^{2j\omega}-e^{-2j\omega} = 2j\sin(2\omega) \end{aligned}$$

**5.3.** Determine the Fourier transform for $-\pi \le \omega < \pi$ in the case of each of the following periodic signals:

**(a)** $\sin(\frac{\pi}{3}n + \frac{\pi}{4})$ **(b)** $2 + \cos(\frac{\pi}{6}n + \frac{\pi}{8})$

We note from Section 5.2 that a periodic signal $x[n]$ with Fourier series representation

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$$

has a Fourier transform

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - \frac{2\pi k}{N}\right).$$

(a) Consider the signal $x_1[n] = \sin(\frac{\pi}{3}n + \frac{\pi}{4})$. We note that the fundamental period of the signal $x_1[n]$ is $N = 6$. The signal may be written as

$$x_1[n] = (1/2j)e^{j(\frac{\pi}{3}n+\frac{\pi}{4})} - (1/2j)e^{-j(\frac{\pi}{3}n+\frac{\pi}{4})} = (1/2j)e^{j\frac{\pi}{4}}e^{j\frac{2\pi}{6}n} - (1/2j)e^{-j\frac{\pi}{4}}e^{-j\frac{2\pi}{6}n}.$$

From this, we obtain the non-zero Fourier series coefficients $a_k$ of $x_1[n]$ in the range $-2 \le k \le 3$ as

$$a_1 = (1/2j)e^{j\frac{\pi}{4}}, \qquad a_{-1} = -(1/2j)e^{-j\frac{\pi}{4}}.$$

Therefore, in the range $-\pi \le \omega \le \pi$, we obtain

$$\begin{aligned} X(e^{j\omega}) &= 2\pi a_1 \delta(\omega - \frac{2\pi}{6}) + 2\pi a_{-1}\delta(\omega + \frac{2\pi}{6}) \\ &= (\pi/j)\{e^{j\pi/4}\delta(\omega - 2\pi/6) - e^{-j\pi/4}\delta(\omega + 2\pi/6)\} \end{aligned}$$

(b) Consider the signal $x_2[n] = 2 + \cos(\frac{\pi}{6}n + \frac{\pi}{8})$. We note that the fundamental period of the signal $x_1[n]$ is $N = 12$. The signal may be written as

$$x_1[n] = 2 + (1/2)e^{j(\frac{\pi}{6}n+\frac{\pi}{8})} + (1/2)e^{-j(\frac{\pi}{6}n+\frac{\pi}{8})} = 2 + (1/2)e^{j\frac{\pi}{8}}e^{j\frac{2\pi}{12}n} + (1/2)e^{-j\frac{\pi}{8}}e^{-j\frac{2\pi}{12}n}.$$

From this, we obtain the non-zero Fourier series coefficients $a_k$ of $x_2[n]$ in the range $-5 \le k \le 6$ as

$$a_0 = 2, \qquad a_1 = (1/2)e^{j\frac{\pi}{8}}, \qquad a_{-1} = (1/2)e^{-j\frac{\pi}{8}}.$$

Therefore, in the range $-\pi \le \omega \le \pi$, we obtain

$$X(e^{j\omega}) = 2\pi a_0 \delta(\omega) + 2\pi a_1 \delta(\omega - \frac{2\pi}{12}) + 2\pi a_{-1}\delta(\omega + \frac{2\pi}{12})$$

**5.6.** Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transforms of the following signals in terms of $X(e^{j\omega})$. You may use the Fourier transform properties listed in Table 5.1.

**(a)** $x_1[n] = x[1-n] + x[-1-n]$

**(b)** $x_2[n] = \frac{x^*[-n]+x[n]}{2}$

**(c)** $x_3[n] = (n-1)^2 x[n]$

Throughout this problem, we assume that

$$x[n] \overset{FT}{\longleftrightarrow} X_1(e^{j\omega}).$$

(a) Using the time reversal property (Sec. 5.3.6), we have

$$x[-n] \overset{FT}{\longleftrightarrow} X(e^{-j\omega})$$

Using the time shifting property (Sec. 5.3.3) on this, we have

$$x[-n+1] \overset{FT}{\longleftrightarrow} e^{-j\omega n} X(e^{-j\omega}) \quad \text{and} \quad x[-n-1] \overset{FT}{\longleftrightarrow} e^{j\omega n} X(e^{-j\omega})$$

Therefore,

$$x_1[n] = x[-n+1] + x[-n-1] \overset{FT}{\longleftrightarrow} e^{-j\omega n} X(e^{-j\omega}) + e^{j\omega n} X(e^{-j\omega})$$

$$\overset{FT}{\longleftrightarrow} 2X(e^{-j\omega}) \cos\omega$$

(b) Using the time reversal property (Sec. 5.3.6), we have

$$x[-n] \overset{FT}{\longleftrightarrow} X(e^{-j\omega})$$

Using the conjugation property on this, we have

$$x^*[-n] \overset{FT}{\longleftrightarrow} X^*(e^{j\omega})$$

Therefore,

$$x_2[n] = (1/2)(x^*[-n] + x[n]) \overset{FT}{\longleftrightarrow} (1/2)(X(e^{j\omega}) + X^*(e^{j\omega}))$$

$$\overset{FT}{\longleftrightarrow} \mathcal{R}e\{X(e^{j\omega})\}$$

(c) Using the differentiation in frequency property (Sec. 5.3.8), we have

$$nx[n] \overset{FT}{\longleftrightarrow} j\frac{dX(e^{j\omega})}{d\omega}$$

Using the same property a second time,

$$n^2 x[n] \overset{FT}{\longleftrightarrow} -\frac{d^2X(e^{j\omega})}{d\omega^2}$$

Therefore,

$$x_3[n] = n^2x[n] - 2nx[n] + 1 \overset{FT}{\longleftrightarrow} -\frac{d^2X(e^{j\omega})}{d\omega^2} - 2j\frac{dX(e^{j\omega})}{d\omega} + X(e^{j\omega})$$

**5.8.** Use Tables 5.1 and 5.2 to help determine $x[n]$ when its Fourier transform is

$$X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}\left(\frac{\sin\frac{3}{2}\omega}{\sin\frac{\omega}{2}}\right) + 5\pi\delta(\omega), \quad -\pi < \omega \le \pi$$

Consider the signal

$$x_1[n] = \begin{cases} 1, & |n| \le 1 \\ 0, & |n| > 1 \end{cases}$$

From Table 5.2, we know that

$$x_1[n] \stackrel{FT}{\longleftrightarrow} X_1(e^{j\omega}) = \frac{\sin(3\omega/2)}{\sin(\omega/2)}$$

Using the accumulation property (Table 5.1, Property 5.3.5), we have

$$\sum_{k=-\infty}^{n} x_1[k] \stackrel{FT}{\longleftrightarrow} \frac{1}{1-e^{-j\omega}}X_1(e^{j\omega}) + \pi X_1(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k).$$

Therefore, in the range $-\pi < \omega \le \pi$,

$$\sum_{k=-\infty}^{n} x_1[k] \stackrel{FT}{\longleftrightarrow} \frac{1}{1-e^{-j\omega}}X_1(e^{j\omega}) + 3\pi\delta(\omega).$$

Also, in the range $-\pi < \omega \le \pi$,

$$1 \stackrel{FT}{\longleftrightarrow} 2\pi\delta(\omega)$$

Therefore, in the range $-\pi < \omega \le \pi$,

$$x[n] = 1 + \sum_{k=-\infty}^{n} x_1[k] \stackrel{FT}{\longleftrightarrow} \frac{1}{1-e^{-j\omega}}X_1(e^{j\omega}) + 5\pi\delta(\omega).$$

The signal $x[n]$ has the desired Fourier transform. We may express $x[n]$ mathematically as

$$x[n] = 1 + \sum_{k=-\infty}^{n} x_1[k] = \begin{cases} 1, & n \le -2 \\ n+3, & -1 \le n \le 1 \\ 4, & n \ge 2 \end{cases}$$

**5.9.** The following four facts are given about a real signal $x[n]$ with Fourier transform $X(e^{jw})$:

1. $x[n] = 0$ for $n > 0$.
2. $x[0] > 0$.
3. $\mathcal{Im}\{X(e^{j\omega})\} = \sin\omega - \sin 2\omega$.
4. $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega = 3$.

Determine $x[n]$.

From Property 5.3.4 in Table 5.1, we know that for a real signal $x[n]$,

$$\mathcal{O}d\{x[n]\} \overset{FT}{\longleftrightarrow} j\mathcal{I}m\{X(e^{j\omega})\}$$

From the given information,

$$\begin{aligned} j\mathcal{I}m\{X(e^{j\omega})\} &= j\sin\omega - j\sin 2\omega \\ &= (1/2)(e^{j\omega} - e^{-j\omega} - e^{2j\omega} + e^{-2j\omega}) \end{aligned}$$

Therefore,

$$\mathcal{O}d\{x[n]\} = \mathcal{IFT}\{j\mathcal{I}m\{X(e^{j\omega})\}\} = (1/2)(\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2])$$

We also know that

$$\mathcal{O}d\{x[n]\} = \frac{x[n] - x[-n]}{2}$$

and that $x[n] = 0$ for $n > 0$. Therefore,

$$x[n] = 2\mathcal{O}d\{x[n]\} = \delta[n+1] - \delta[n+2], \qquad \text{for } n < 0.$$

Now we only have to find $x[0]$. Using Parseval's relation, we have

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(e^{j\omega})|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2.$$

From the given information, we can write

$$3 = (x[0])^2 + \sum_{n=-\infty}^{-1} |x[n]|^2 = (x[0])^2 + 2$$

This gives $x[0] = \pm 1$. But since we are given that $x[0] > 0$, we conclude that $x[0] = 1$. Therefore,

$$x[n] = \delta[n] + \delta[n+1] - \delta[n+2].$$

**5.13.** An LTI system with impulse response $h_1[n] = (\frac{1}{3})^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}.$$

Determine $h_2[n]$.

When two LTI systems are connected in parallel, the impulse response of the overall system is the sum of the impulse responses of the individual systems. Therefore,

$$h[n] = h_1[n] + h_2[n].$$

Using the linearity property (Table 5.1, Property 5.3.2),

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

Given that $h_1[n] = (1/2)^n u[n]$, we obtain

$$H_1(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}$$

Therefore,

$$H_2(e^{j\omega}) = \frac{-12 + 5^{-j\omega}}{12 - 7e^{-j\omega} + e^{-2j\omega}} - \frac{1}{1 - \frac{1}{2}e^{-j\omega}} = \frac{-2}{1 - \frac{1}{4}e^{-j\omega}}.$$

Taking the inverse Fourier transform,

$$h_2[n] = -2\left(\frac{1}{4}\right)^n u[n].$$

**5.18.** Given the fact that

$$a^{|n|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{1-a^2}{1-2a\cos\omega + a^2}, \quad |a| < 1,$$

use duality to determine the Fourier series coefficients of the following continuous-time signal with period $T = 1$:

$$x(t) = \frac{1}{5 - 4\cos(2\pi t)}.$$

Knowing that

$$\left(\frac{1}{2}\right)^{|n|} \overset{FT}{\longleftrightarrow} \frac{1-\frac{1}{4}}{1-\cos\omega+\frac{1}{4}} = \frac{3}{5-4\cos\omega},$$

we may use the Fourier transform analysis equation to write

$$\frac{3}{5-4\cos\omega} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|n|} e^{-j\omega n}$$

Putting $\omega = -2\pi t$ in this equation, and replacing the variable $n$ by the variable $k$

$$\frac{1}{5-4\cos(2\pi t)} = \sum_{k=-\infty}^{\infty} \frac{1}{3}\left(\frac{1}{2}\right)^{|k|} e^{j2\pi kt}.$$

By comparing this with the continuous-time Fourier series synthesis equation, it is immediately apparent that $a_k = \frac{1}{3}\left(\frac{1}{2}\right)^{|k|}$ are the Fourier series coefficients of the signal $1/(5 - 4\cos(2\pi t))$.

**5.22.** The following are the Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.

**(a)** $X(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi, 0 \le |\omega| < \frac{\pi}{4} \end{cases}$

**(b)** $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$

**(c)** $X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \le \omega \le \pi$

**(d)** $X(e^{j\omega}) = \cos^2\omega + \sin^2 3\omega$

**(e)** $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty}(-1)^k\delta(\omega - \frac{\pi}{2}k)$

**(f)** $X(e^{j\omega}) = \frac{e^{-j\omega} - \frac{1}{5}}{1 - \frac{1}{5}e^{-j\omega}}$

**(g)** $X(e^{j\omega}) = \frac{1 - \frac{1}{3}e^{-j\omega}}{1 - \frac{1}{4}e^{-j\omega} - \frac{1}{8}e^{-2j\omega}}$

**(h)** $X(e^{j\omega}) = \frac{1 - (\frac{1}{3})^6 e^{-j6\omega}}{1 - \frac{1}{3}e^{-j\omega}}$

(a) Using the Fourier transform synthesis eq. (5.8), we obtain

$$\begin{aligned} x[n] &= \frac{1}{2\pi}\int_{-3\pi/4}^{-\pi/4} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\pi/4}^{3\pi/4} e^{j\omega n} d\omega \\ &= \frac{1}{\pi n}[\sin(3\pi n/4) - \sin(\pi n/4)] \end{aligned}$$

(b) Comparing the given Fourier transform with the analysis eq. (5.8), we obtain

$$x[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] - 4\delta[n-3] + \delta[n-10].$$

(c) Using the Fourier transform synthesis eq. (5.8), we obtain

$$\begin{aligned} x[n] &= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega/2} e^{j\omega n} d\omega \\ &= \frac{(-1)^{n+1}}{\pi(n - \frac{1}{2})} \end{aligned}$$

(d) The given Fourier transform is

$$\begin{aligned} X(e^{j\omega}) &= \cos^2\omega + \sin^2(3\omega) \\ &= \frac{1 + \cos(2\omega)}{2} + \frac{1 - \cos(3\omega)}{2} \\ &= 1 + \frac{1}{4}e^{2j\omega} + \frac{1}{4}e^{-2j\omega} + -\frac{1}{4}e^{3j\omega} - \frac{1}{4}e^{-3j\omega} \end{aligned}$$

Comparing the given Fourier transform with the analysis eq. (5.8), we obtain

$$x[n] = \delta[n] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n+2] - \frac{1}{4}\delta[n-3] - \frac{1}{4}\delta[n+3].$$

(e) This is the Fourier transform of a periodic signal with fundamental frequency $\pi/2$. Therefore, its fundamental period is 4. Also, the Fourier series coefficients of this signal are $a_k = (-1)^k$. Therefore, the signal is given by

$$x[n] = \sum_{k=0}^{3}(-1)^k e^{jk(\pi/2)n} = 1 - e^{j\pi n/2} + e^{j\pi n} - e^{j3\pi n/2}.$$

(f) The given Fourier transform may be written as

$$\begin{aligned} X(e^{j\omega}) &= e^{-j\omega}\sum_{n=0}^{\infty}(1/5)^n e^{-j\omega n} - (1/5)\sum_{n=0}^{\infty}(1/5)^n e^{-j\omega n} \\ &= 5\sum_{n=1}^{\infty}(1/5)^n e^{-j\omega n} - (1/5)\sum_{n=0}^{\infty}(1/5)^n e^{-j\omega n} \end{aligned}$$

Comparing each of the two terms in the right-hand side of the above equation with the Fourier transform analysis eq. (5.9) we obtain

$$x[n] = \left(\frac{1}{5}\right)^{n-1} u[n-1] - \left(\frac{1}{5}\right)^{n+1} u[n].$$

(g) The given Fourier transform may be written as

$$X(e^{j\omega}) = \frac{2/9}{1 - \frac{1}{2}e^{-j\omega}} + \frac{7/9}{1 + \frac{1}{4}e^{-j\omega}}.$$

Therefore,

$$x[n] = \frac{2}{9}\left(\frac{1}{2}\right)^n u[n] + \frac{7}{9}\left(-\frac{1}{4}\right)^n u[n].$$

(h) The given Fourier transform may be written as

$$X(e^{j\omega}) = 1 + \frac{1}{3}e^{-j\omega} + \frac{1}{3^2}e^{-j2\omega} + \frac{1}{3^3}e^{-j3\omega} + \frac{1}{3^4}e^{-j4\omega} + \frac{1}{3^5}e^{-j5\omega}.$$

Comparing the given Fourier transform with the analysis eq. (5.8), we obtain

$$x[n] = \delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{9}\delta[n-2] + \frac{1}{27}\delta[n-3] + \frac{1}{81}\delta[n-4] + \frac{1}{243}\delta[n-5].$$

**5.23.** Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ depicted in Figure P5.23. Perform the following calculations without explicitly evaluating $X(e^{j\omega})$:

**(a)** Evaluate $X(e^{j0})$.

**(b)** Find $\sphericalangle X(e^{j\omega})$.

**(c)** Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$.

**(d)** Find $X(e^{j\pi})$.

**(e)** Determine and sketch the signal whose Fourier transform is $\mathcal{R}e\{x(\omega)\}$.

**(f)** Evaluate:

**(i)** $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

**(ii)** $\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega$

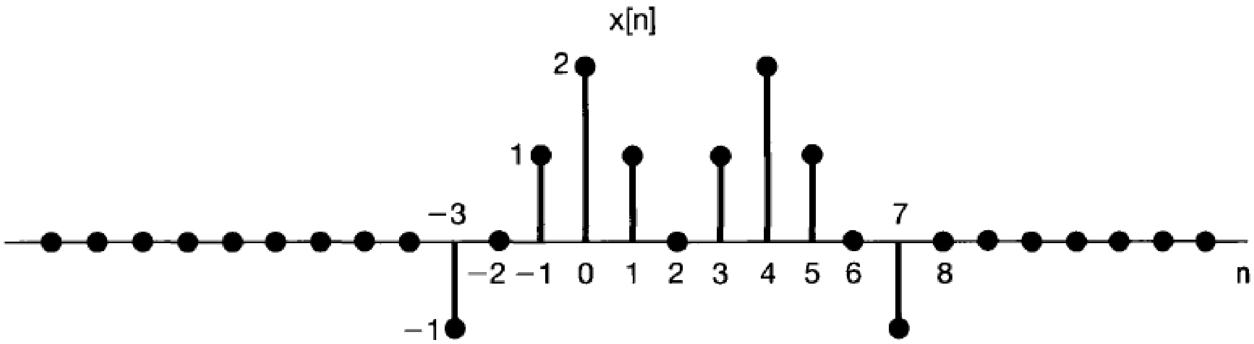


**Fig P5.23**

(a) We have from eq. (5.9)

$$X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n] = 6.$$

(b) Note that $y[n] = x[n+2]$ is an even signal. Therefore, $Y(e^{j\omega})$ is real and even. This implies that $\sphericalangle Y(e^{j\omega}) = 0$. Furthermore, from the time shifting property of the Fourier transform we have $Y(e^{j\omega}) = e^{j2\omega}X(e^{j\omega})$. Therefore, $\sphericalangle X(e^{j\omega}) = e^{-j2\omega}$.

(c) We have from eq. (5.8)

$$2\pi x[0] = \int_{-\pi}^{\pi} X(e^{j\omega})d\omega.$$

Therefore,

$$\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 4\pi.$$

(d) We have from eq. (5.9)

$$X(e^{j\pi}) = \sum_{n=-\infty}^{\infty} x[n](-1)^n = 2.$$

(e) From Table 5.1, we have

$$\mathcal{E}v\{x[n]\} \stackrel{FT}{\longleftrightarrow} \mathcal{R}e\{X(e^{j\omega})\}.$$

Therefore, the desired signal is $\mathcal{E}v\{x[n]\} = (x[n]+x[-n])/2$. This is as shown in Figure S5.23.

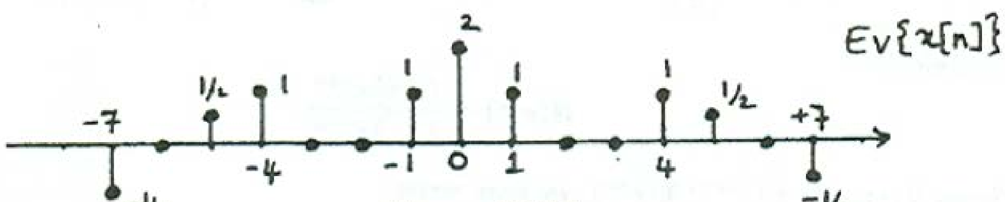


Figure S5.23

(f) (i) From Parseval's theorem we have

$$\int_{-\infty}^{\infty} |X(e^{j\omega})|^2 = 2\pi \sum_{n=-\infty}^{\infty} |x[n]|^2 = 28\pi.$$

(ii) Using the differentiation in frequency property of the Fourier transform we obtain

$$nx[n] \stackrel{FT}{\longleftrightarrow} j\frac{dX(e^{j\omega})}{d\omega}.$$

Again using Parseval's theorem, we obtain

$$\int_{-\infty}^{\infty} |\frac{dX(e^{j\omega})}{d\omega}|^2 = 2\pi \sum_{n=-\infty}^{\infty} |n|^2|x[n]|^2 = 316\pi.$$

**5.29.** **(a)** Consider a discrete-time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

Use Fourier transforms to determine the response to each of the following input signals:

**(i)** $x[n] = (\frac{3}{4})^n u[n]$

**(ii)** $x[n] = (n+1)(\frac{1}{4})^n u[n]$

**(iii)** $x[n] = (-1)^n$

**(b)** Suppose that

$$h[n] = \left[\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)\right] u[n].$$

Use Fourier transforms to determine the response to each of the following inputs:

**(i)** $x[n] = (\frac{1}{2})^n u[n]$

**(ii)** $x[n] = \cos(\pi n/2)$

**(c)** Let $x[n]$ and $h[n]$ be signals with the following Fourier transforms:

$$X(e^{j\omega}) = 3e^{j\omega} + 1 - e^{-j\omega} + 2e^{-j3\omega},$$

$$H(e^{j\omega}) = -e^{j\omega} + 2e^{-2j\omega} + e^{j4\omega}.$$

Determine $y[n] = x[n] * h[n]$.

(a) Let the output of the system be $y[n]$. We know that

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}).$$

In this part of the problem

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}.$$

(i) We have

$$X(e^{j\omega}) = \frac{1}{1-\frac{3}{4}e^{-j\omega}}.$$

Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[\frac{1}{1-\frac{3}{4}e^{-j\omega}}\right]\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{-2}{1-\frac{1}{2}e^{-j\omega}} + \frac{3}{1-\frac{3}{4}e^{-j\omega}} \end{aligned}$$

Taking the inverse Fourier transform, we obtain

$$y[n] = 3\left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{1}{2}\right)^n u[n].$$

(ii) We have

$$X(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}.$$

Therefore,

$$\begin{aligned} Y(e^{j\omega}) &= \left[\frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}\right]\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] \\ &= \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}} - \frac{3}{(1-\frac{1}{4}e^{-j\omega})^2} \end{aligned}$$