به نام خدا

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2.3. Consider an input $x[n]$ and a unit impulse response $h[n]$ given by

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2],$$

$$h[n] = u[n+2].$$

Determine and plot the output $y[n] = x[n] * h[n]$.

Let us define the signals

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$$

and

$$h_1[n] = u[n].$$

We note that

$$x[n] = x_1[n-2] \quad \text{and} \quad h[n] = h_1[n+2]$$

Now,

$$\begin{aligned} y[n] &= x[n] * h[n] = x_1[n-2] * h_1[n+2] \\ &= \sum_{k=-\infty}^{\infty} x_1[k-2]h_1[n-k+2] \end{aligned}$$

By replacing k with $m+2$ in the abovr summation, we obtain

$$y[n] = \sum_{m=-\infty}^{\infty} x_1[m]h_1[n-m] = x_1[n] * h_1[n]$$

Using the results of Example 2.1 in the text book, we may write

$$y[n] = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right] u[n]$$

2.4. Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1, & 3 \leq n \leq 8 \\ 0, & \text{otherwise} \end{cases},$$

$$h[n] = \begin{cases} 1, & 4 \leq n \leq 15 \\ 0, & \text{otherwise} \end{cases}.$$

We know that

$$y[n] = x[n] * h[n] = \sum_{k=\infty}^{\infty} x[k]h[n-k]$$

The signals $x[n]$ and $y[n]$ are as shown in Figure S2.4. From this figure, we see that the above summation reduces to

$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$

This gives

$$y[n] = \begin{cases} n-6, & 7 \leq n \leq 11 \\ 6, & 12 \leq n \leq 18 \\ 24-n, & 19 \leq n \leq 23 \\ 0, & \text{otherwise} \end{cases}$$

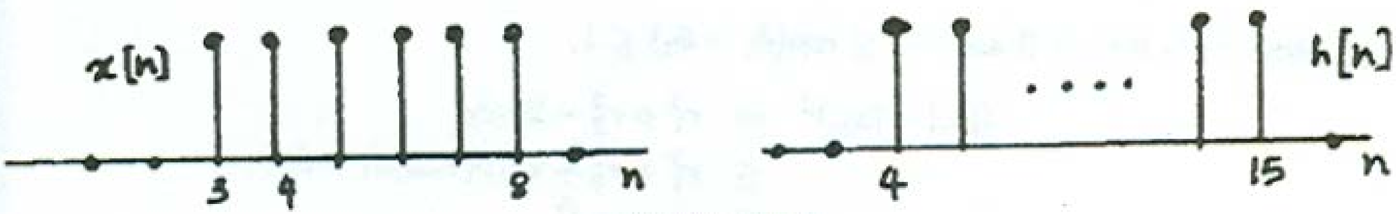


Figure S2.4

2.21. Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

(d) $x[n]$ and $h[n]$ are as in Figure P2.21.

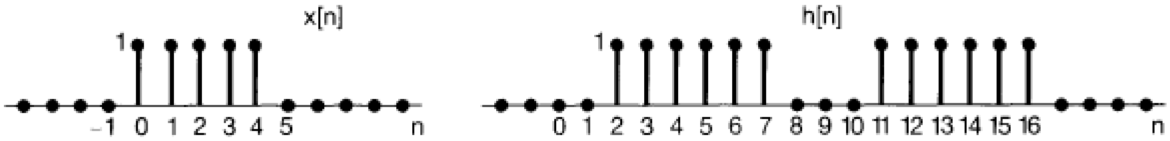
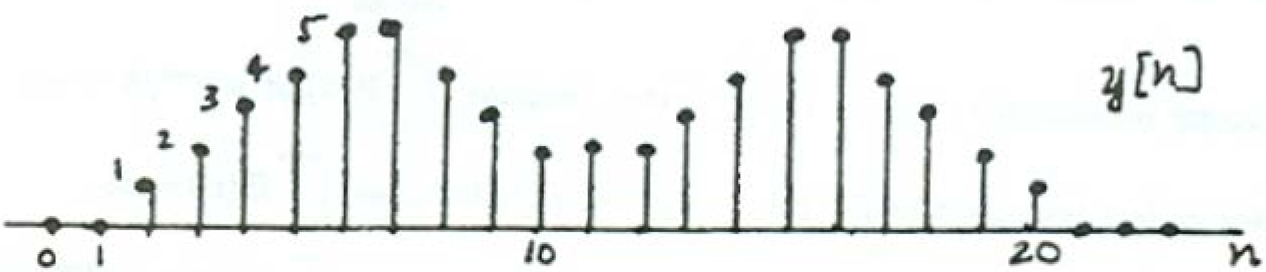


Figure P2.21

The desired convolution is

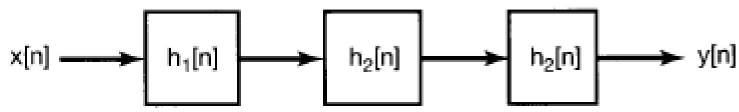
$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\ &= x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3] + x[4]h[n-4] \\ &= h[n] + h[n-1] + h[n-2] + h[n-3] + h[n-4]. \end{aligned}$$



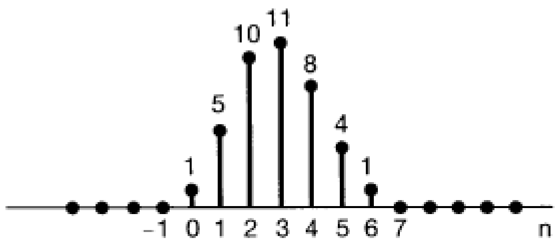
2.24. Consider the cascade interconnection of three causal LTI systems, illustrated in Figure P2.24(a). The impulse response $h_2[n]$ is

$$h_2[n] = u[n] - u[n-2],$$

and the overall impulse response is as shown in Figure P2.24(b).



(a)



(b)

Figure P2.24

(a) Find the impulse response $h_1[n]$.
(b) Find the response of the overall system to the input

$$x[n] = \delta[n] - \delta[n-1].$$

(a) We are given that $h_2[n] = \delta[n] + \delta[n-1]$. Therefore,

$$h_2[n] * h_2[n] = \delta[n] + 2\delta[n-1] + \delta[n-2].$$

Since

$$h[n] = h_1[n] * [h_2[n] * h_2[n]],$$

we get

$$h[n] = h_1[n] + 2h_1[n-1] + h_1[n-2].$$

Therefore,

$$\begin{aligned}
h[0] &= h_1[0] & \Rightarrow \quad & h_1[0] = 1, \\
h[1] &= h_1[1] + 2h_1[0] & \Rightarrow \quad & h_1[1] = 3, \\
h[2] &= h_1[2] + 2h_1[1] + h_1[0] & \Rightarrow \quad & h_1[2] = 3, \\
h[3] &= h_1[3] + 2h_1[2] + h_1[1] & \Rightarrow \quad & h_1[3] = 2, \\
h[4] &= h_1[4] + 2h_1[3] + h_1[2] & \Rightarrow \quad & h_1[4] = 1 \\
h[5] &= h_1[5] + 2h_1[4] + h_1[3] & \Rightarrow \quad & h_1[5] = 0.
\end{aligned}$$

$h_1[n] = 0$ for $n < 0$ and $n \geq 5$.

(b) In this case,

$$y[n] = x[n] * h[n] = h[n] - h[n-1].$$