به نام خدا

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مهلت تحویل: ۱ خرداد ۱۳۹۸

5.2. Use the Fourier transform analysis equation (5.9) to calculate the Fourier transforms of:

(a) $\delta[n-1]+\delta[n+1]$ **(b)** $\delta[n+2]-\delta[n-2]$

Sketch and label one period of the magnitude of each Fourier transform.

5.3. Determine the Fourier transform for $-\pi \leq \omega < \pi$ in the case of each of the following periodic signals:

(a) $\sin(\frac{\pi}{3}n+\frac{\pi}{4})$ **(b)** $2+\cos(\frac{\pi}{6}n+\frac{\pi}{8})$

5.6. Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transforms of the following signals in terms of $X(e^{j\omega})$. You may use the Fourier transform properties listed in Table 5.1.

(a) $x_1[n] = x[1-n]+x[-1-n]$

(b) $x_2[n] = \frac{x^*[-n]+x[n]}{2}$

(c) $x_3[n] = (n-1)^2 x[n]$

5.8. Use Tables 5.1 and 5.2 to help determine $x[n]$ when its Fourier transform is

$$X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}\left(\frac{\sin\frac{3}{2}\omega}{\sin\frac{\omega}{2}}\right)+5\pi\delta(\omega), \quad -\pi < \omega \leq \pi$$

5.9. The following four facts are given about a real signal $x[n]$ with Fourier transform $X(e^{jw})$:

1. $x[n] = 0$ for $n > 0$.
2. $x[0] > 0$.
3. $\mathcal{Im}\{X(e^{j\omega})\} = \sin\omega - \sin 2\omega$.
4. $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3$.

Determine $x[n]$.

5.13. An LTI system with impulse response $h_1[n] = (\frac{1}{3})^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}.$$

Determine $h_2[n]$.

5.18. Given the fact that

$$a^{|n|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{1 - a^2}{1 - 2a\cos\omega + a^2}, \quad |a| < 1,$$

use duality to determine the Fourier series coefficients of the following continuous-time signal with period $T = 1$:

$$x(t) = \frac{1}{5 - 4\cos(2\pi t)}.$$

5.22. The following are the Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.

(a) $X(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi, 0 \le |\omega| < \frac{\pi}{4} \end{cases}$

(b) $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$

(c) $X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \le \omega \le \pi$

(d) $X(e^{j\omega}) = \cos^2\omega + \sin^2 3\omega$

(e) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty}(-1)^k\delta(\omega - \frac{\pi}{2}k)$

(f) $X(e^{j\omega}) = \frac{e^{-j\omega} - \frac{1}{5}}{1 - \frac{1}{5}e^{-j\omega}}$

(g) $X(e^{j\omega}) = \frac{1 - \frac{1}{3}e^{-j\omega}}{1 - \frac{1}{4}e^{-j\omega} - \frac{1}{8}e^{-2j\omega}}$

(h) $X(e^{j\omega}) = \frac{1 - (\frac{1}{3})^6 e^{-j6\omega}}{1 - \frac{1}{3}e^{-j\omega}}$

5.23. Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ depicted in Figure P5.23. Perform the following calculations without explicitly evaluating $X(e^{j\omega})$:

(a) Evaluate $X(e^{j0})$.

(b) Find $\measuredangle X(e^{j\omega})$.

(c) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$.

(d) Find $X(e^{j\pi})$.

(e) Determine and sketch the signal whose Fourier transform is $\mathcal{R}e\{x(\omega)\}$.

(f) Evaluate:

(i) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

(ii) $\int_{-\pi}^{\pi} \left| \frac{dX(e^{j\omega})}{d\omega} \right|^2 d\omega$

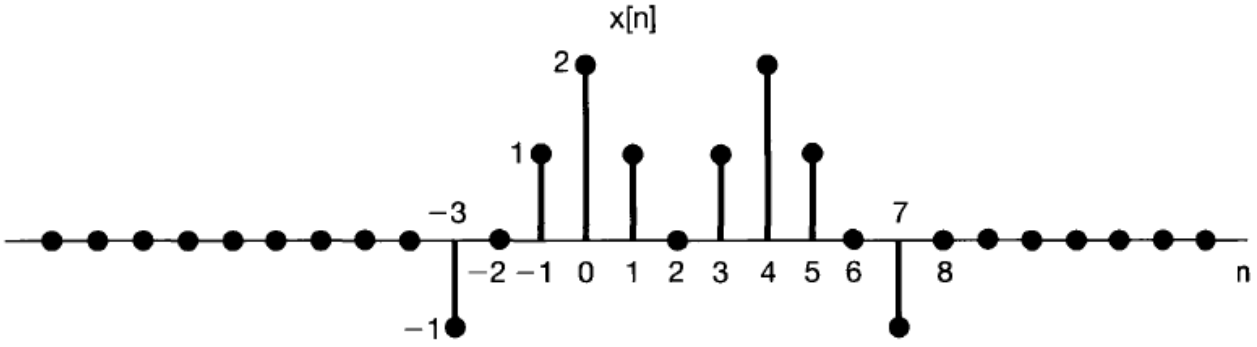

Fig P5.23

5.29. **(a)** Consider a discrete-time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

Use Fourier transforms to determine the response to each of the following input signals:

(i) $x[n] = (\frac{3}{4})^n u[n]$

(ii) $x[n] = (n+1)(\frac{1}{4})^n u[n]$

(iii) $x[n] = (-1)^n$

(b) Suppose that

$$h[n] = \left[\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)\right] u[n].$$

Use Fourier transforms to determine the response to each of the following inputs:

(i) $x[n] = (\frac{1}{2})^n u[n]$

(ii) $x[n] = \cos(\pi n/2)$

(c) Let $x[n]$ and $h[n]$ be signals with the following Fourier transforms:

$$X(e^{j\omega}) = 3e^{j\omega} + 1 - e^{-j\omega} + 2e^{-j3\omega},$$

$$H(e^{j\omega}) = -e^{j\omega} + 2e^{-2j\omega} + e^{j4\omega}.$$

Determine $y[n] = x[n] * h[n]$.
