به نام خدا

دانشگاه آزاد اسلامی واحد نجف آباد دانشکده مهندسی برق- دکتر فغانی

تکلیف سری چهارم سیگنالها و سیستمها

مهلت تحویل: ۱ خرداد ۱۳۹۸

4.2. Use the Fourier transform analysis equation (4.9) to calculate the Fourier transforms of:

(a) $\delta(t+1)+\delta(t-1)$ **(b)** $\frac{d}{dt}\{u(-2-t)+u(t-2)\}$

Sketch and label the magnitude of each Fourier transform.

4.3. Determine the Fourier transform of each of the following periodic signals:

(a) $\sin(2\pi t+\frac{\pi}{4})$ **(b)** $1+\cos(6\pi t+\frac{\pi}{8})$

4.6. Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may find useful the Fourier transform properties listed in Table 4.1.

(a) $x_1(t) = x(1-t)+x(-1-t)$

(b) $x_2(t) = x(3t-6)$

(c) $x_3(t) = \frac{d^2}{dt^2}x(t-1)$

4.7. For each of the following Fourier transforms, use Fourier transform properties (Table 4.1) to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the inverse of any of the given transforms.

(a) $X_1(j\omega) = u(\omega)-u(\omega-2)$

(b) $X_2(j\omega) = \cos(2\omega)\sin(\frac{\omega}{2})$

(c) $X_3(j\omega) = A(\omega)e^{jB(\omega)}$, where $A(\omega) = (\sin 2\omega)/\omega$ and $B(\omega) = 2\omega+\frac{\pi}{2}$

(d) $X(j\omega) = \sum_{k=-\infty}^{\infty}(\frac{1}{2})^{|k|}\,\delta(\omega-\frac{k\pi}{4})$

4.10. **(a)** Use Tables 4.1 and 4.2 to help determine the Fourier transform of the following signal:

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$$

(b) Use Parseval's relation and the result of the previous part to determine the numerical value of

$$A = \int_{-\infty}^{+\infty} t^2 \left(\frac{\sin t}{\pi t}\right)^4 dt$$

4.12. Consider the Fourier transform pair

$$e^{-|t|} \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{2}{1+\omega^2}.$$

(a) Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$.

(b) Use the result from part (a), along with the duality property, to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}.$$

Hint: See Example 4.13.

4.15. Let $x(t)$ be a signal with Fourier transform $X(j\omega)$. Suppose we are given the following facts:

1. $x(t)$ is real.

2. $x(t) = 0$ for $t \le 0$.

3. $\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{R}e\{X(j\omega)\}e^{j\omega t}\, d\omega = |t|e^{-|t|}$.

Determine a closed-form expression for $x(t)$.

4.18. Find the impulse response of a system with the frequency response

$$H(j\omega) = \frac{(\sin^2(3\omega))\cos\omega}{\omega^2}.$$

4.19. Consider a causal LTI system with frequency response

$$H(j\omega) = \frac{1}{j\omega + 3}.$$

For a particular input $x(t)$ this system is observed to produce the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t).$$

Determine $x(t)$.

4.22. Determine the continuous-time signal corresponding to each of the following transforms.

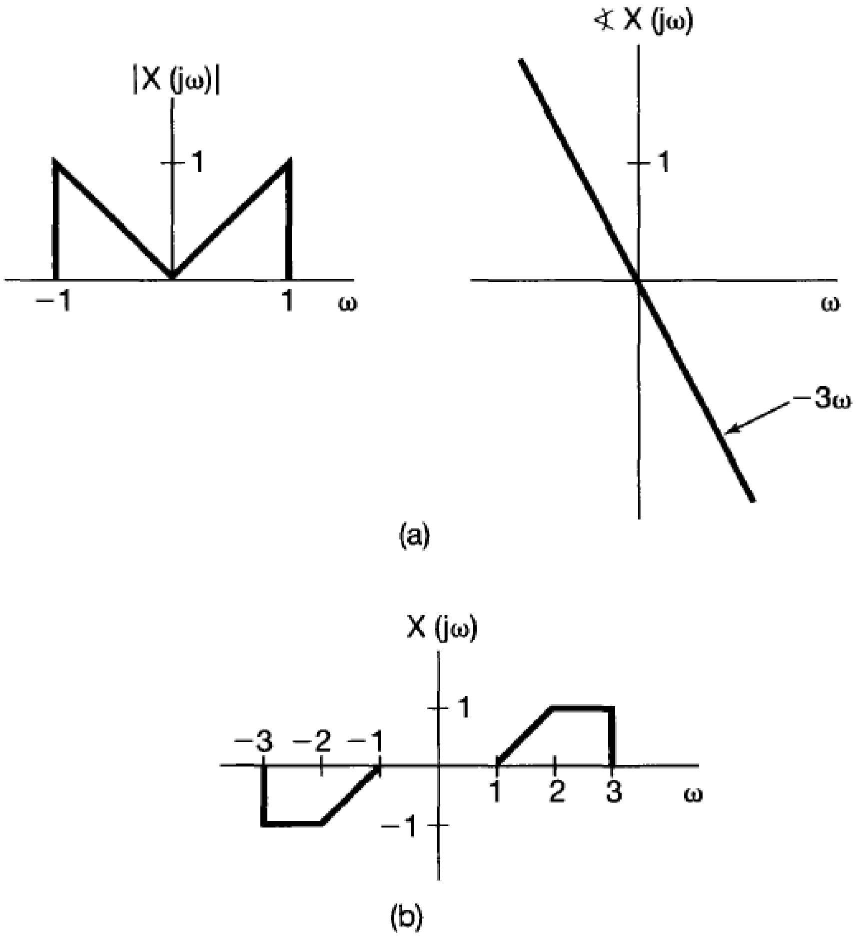

Figure P4.22

(a) $X(j\omega) = \frac{2\sin[3(\omega-2\pi)]}{(\omega-2\pi)}$
(b) $X(j\omega) = \cos(4\omega + \pi/3)$
(c) $X(j\omega)$ as given by the magnitude and phase plots of Figure P4.22(a)
(d) $X(j\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$
(e) $X(j\omega)$ as in Figure P4.22(b)

4.25. Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in Figure P4.25.
(a) Find $\sphericalangle X(j\omega)$.
(b) Find $X(j0)$.
(c) Find $\int_{-\infty}^{\infty} X(j\omega)\, d\omega$.
(d) Evaluate $\int_{-\infty}^{\infty} X(j\omega) \frac{2\sin\omega}{\omega} e^{j2\omega}\, d\omega$.
(e) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega$.
(f) Sketch the inverse Fourier transform of $\mathcal{R}e\{X(j\omega)\}$.
Note: You should perform all these calculations without explicitly evaluating $X(j\omega)$.

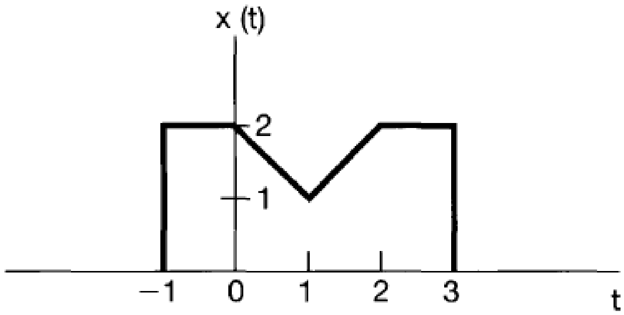

Figure P4.25

4.30. Suppose $g(t) = x(t)\cos t$ and the Fourier transform of the $g(t)$ is

$$G(j\omega) = \begin{cases} 1, & |\omega| \leq 2 \\ 0, & \text{otherwise} \end{cases}.$$

(a) Determine $x(t)$.
(b) Specify the Fourier transform $X_1(j\omega)$ of a signal $x_1(t)$ such that

$$g(t) = x_1(t)\cos\left(\frac{2}{3}t\right).$$

4.31. **(a)** Show that the three LTI systems with impulse responses

$$h_1(t) = u(t),$$

$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t),$$

and

$$h_3(t) = 2te^{-t}u(t)$$

all have the same response to $x(t) = \cos t$.
(b) Find the impulse response of another LTI system with the same response to $\cos t$.

This problem illustrates the fact that the response to $\cos t$ cannot be used to specify an LTI system uniquely.

4.32. Consider an LTI system S with impulse response

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}.$$

Determine the output of S for each of the following inputs:

(a) $x_1(t) = \cos(6t + \frac{\pi}{2})$

(b) $x_2(t) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \sin(3kt)$

(c) $x_3(t) = \frac{\sin(4(t+1))}{\pi(t+1)}$

(d) $x_4(t) = (\frac{\sin 2t}{\pi t})^2$